Stats 2MB3, Tutorial 7

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Confidence Interval

- A 100(1-α)% confidence interval for the mean μ of a normal population when the value of σ is known is given by

$$(\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}})$$

or, equivalently, by $\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$. $z_{\alpha/2}$ is critical value.

Ex 1, page 275

- Consider a normal population distribution with the value of σ known.
- a) What is the confidence level for the interval $\bar{x} \pm 2.81\sigma / \sqrt{n}$
- b) What is the confidence level for the interval $\bar{x} \pm 1.44\sigma / \sqrt{n}$
- c) What value of $z_{\alpha/2}$ results in a confidence level of 99.7%.
- d) Same as question (c) for a confidence interval of 75%.

- a) The critical value $z_{\alpha/2} = 2.81$, which implies that α/2=1-Φ(2.81)=0.0025. Then α=-.005 and the confidence level is 100(1- α)%=99.5%.
- b) $z_{\alpha/2} = 1.44$ implies that α=2[1-Φ(1.44)]=0.15 and the confidence level is 100(1- α)%=85%.
- c) 100(1- α)%=99.7%, then α=0.003 and

  $$z_{\alpha/2} = z_{0.0015} = 2.96$$
- d) 100(1- α)%=75%, then α=0.25, and

  $$z_{\alpha/2} = z_{0.125} = 1.15$$

Ex 3, page 275

- Suppose that a random sample of 50 bottles of a particular brand of cough syrup is selected and alcohol content of each bottle is determined. Let μ denote the average alcohol content for the population of all bottles of the brand under study. Suppose that the resulting 95% confidence interval is (7.8, 9.4).

- a) Would a 90% confidence interval calculated from this same sample have been narrower or wider than the given interval? Explain your reasoning.
- b) There is a 95% chance that μ is between 7.8 and 9.4. Is this statement correct?
- c) We can be highly confident that 95% of all bottles of this type of cough syrup have an alcohol content that is between 7.8 and 9.4. Is this correct?
- d) If the process of selecting a sample of size 50 and then computing the corresponding 95% interval is repeated 100 times, 95 of the resulting interval will include μ. Is this statement correct?

a) The length of the CI is equal to $\frac{2z_{\alpha/2}\sigma}{\sqrt{n}}$.

Since $z_{0.025} = 1.96$ and $z_{0.05} = 1.645$, the 90% confidence interval is narrower.

b) Incorrect! Because μ is not random variable. It can be only enclosed by the interval or not enclosed. Nothing about “chance”.

c) Incorrect! The confidence interval is about the population mean, not every value of the population.

d) Incorrect! In fact, 95 is just the theoretical result based on probability. We can not ensure this phenomenon will definitely occur.

Ex 11, page 276

- Consider the next 1000 95% CI for μ that a statistical consultant will obtain for various clients. Suppose the data sets on which the intervals are based are selected independently of one another. How many of these 1000 intervals do you expect to capture the corresponding value of μ? What is the probability that between 940 and 960 of these intervals contain the corresponding value of μ?

- Set Y as the number of intervals capture μ. Then Y is following binomial distribution with parameters $n=1000$ and $p=0.95$.
- $E(Y)=np=950$, $Var(Y)=np(1-p)=47.5$ and $sd(Y)=6.892$.
- Then using the central limit theorem and approximation modification of discrete distribution, we have

$P(940 \le Y \le 960)=P(939.5 \le Y \le 960.5)$

$=P(-1.52 \le Z \le 1.52)=0.8714.$